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# DISPLACEMENT BASED MULTILEVEL STRUCTURAL OPTIMIZATION

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**ABSTRACT**

In the complex environment of true multidisciplinary design optimization (MDO), efficiency is one of the most desirable attributes of any approach. In the present research, a new and highly efficient methodology for the MDO subset of structural optimization is proposed and detailed, i.e., for the weight minimization of a given structure under size, strength, and displacement constraints. Specifically, finite element based multilevel optimization of structures is performed. In the system level optimization, the design variables are the coefficients of assumed polynomially based global displacement functions, and the load unbalance resulting from the solution of the global stiffness equations is minimized. In the subsystems level optimizations, the weight of each element is minimized under the action of stress constraints, with the cross sectional dimensions as design variables. The approach is expected to prove very efficient since the design task is broken down into a large number of small and efficient subtasks, each with a small number of variables, which are amenable to parallel computing.

## Introduction

Multidisciplinary design optimization (MDO) is expected to play a major role in the competitive advanced transportation industries of tomorrow, such as in the design of commercial and military aircraft and spacecraft, of high speed trains and automobiles, etc. All of these vehicles require maximum performance, e.g., speed, payload capacity, safety, at minimum weight to keep fuel consumption low and conserve resources. Here, multidisciplinary design optimization (MDO) can deliver mathematically based design tools to accomplish the task of obtaining minimum weight with optimum performance based on the constraints of many disciplines, such as structures, aerodynamics, controls, performance, etc. Although some applications of MDO

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are beginning to surface now, the key to the more widespread use of this technology centers around the efficiency improvement of the available MDO approaches.

This aspect is investigated here for the MDO subset of structural optimization, i.e., for the weight minimization of a given structure under size, strength, and displacement constraints. The approach can easily be expanded to include additional structural constraints (buckling, free and forced vibration, etc.) or other disciplines (passive and active controls, aerodynamics, fatigue and fracture, etc.).

## Methodology

For the present investigation, finite element based optimization of structural systems (initially, of trusses, beams, and frames; later, of more realistic thin-walled structures) is proposed and performed in a multilevel approach with a single system level optimization and multiple subsystems level optimizations as outlined in the following (Table 1).

### System Level Analysis and Optimization

The approach uses a system level analysis and optimization in which the design variables are the coefficients of a smooth deflection curve, a deflection surface, or a three-dimensional displacement distribution. When the stiffness equations are solved based on this approximate displacement distribution, the resulting nodal forces are unbalanced, and a square measure of this load is minimized. Constraints are placed on the deflection amplitudes to avoid excessive errors in the displacement field coefficients and on the weight of the structure as a function of these same coefficients.

### Subsystems Level Analyses and Optimizations

In the subsystems level analyses and optimizations, the weight of each element is minimized under the action of stress constraints. These are applied at the nodal locations of the elements and are based on the element forces, which are calculated from the stiffness equations using the prescribed displacements from the system level optimization. In these local optimizations, the element cross sectional dimensions are used as the design variables.

### Constraint Evaluation

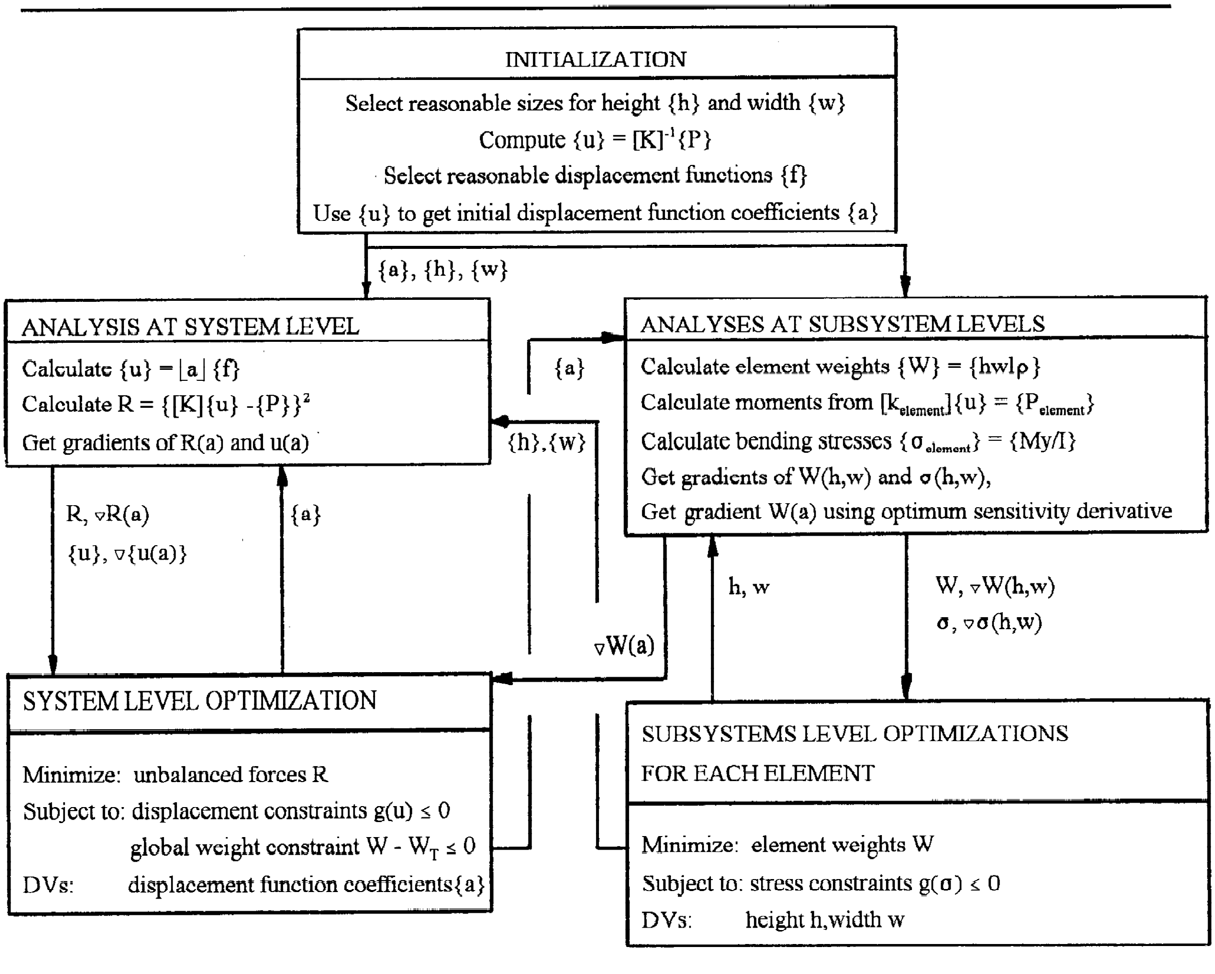
All constraints are straight forward in their development and evaluation with the exception of the weight constraint as a function of the displacement field coefficients in the system level optimization. This constraint needs to be evaluated by means of the optimum sensitivity approach, developed by Sobieszczanski-Sobieski, Barthelemy, and Riley [1]. It yields values of the derivatives of the objective function at the optimum, in this case the weights in the subsystems level optimizations, and of the design variables with respect to those physical quantities, here the prescribed displacements and their coefficients in the system level optimization, which were kept constant as problem parameters during the local optimization stage.

### Efficiency of the Approach

The proposed approach is expected to be very efficient, especially for complex structures, since the design task is broken down into two parts:

1. A system level optimization which is based on only a limited number of design variables since the displacement shape of a continuous structure is, in general, smooth and can be approximated by only a

Table 1. Methodology for Regular Approach on Beam Model



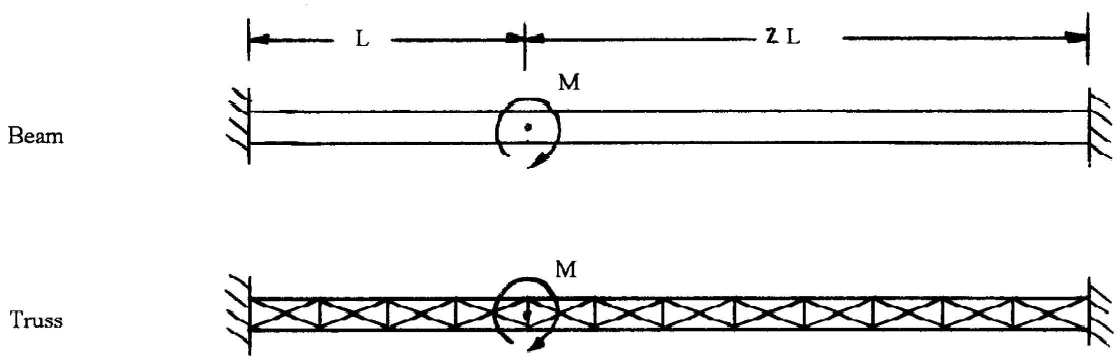




Figure 1. Initial Optimization Models

small number of coefficients as they are multiplied by either polynomial or trigonometric approximating functions.

2. A large number of small and efficiently handled subsystems level optimizations, each with only a small number of design variables. This partitioning will also allow for the use of parallel computing. Initially, the system and subsystems level optimizations will be sent to two different processors. Ultimately, it is expected that the subsystems level optimizations will be carried out separately for each element in a massively parallel manner on separate processors.

## Alternate Approaches

### Alternate System Level Approach

In the system level optimization, an alternate approach replaces the displacement coefficients as design variables by small variations of the displacements themselves at a limited number of nodal points. The initial displacements are computed by the system finite element analysis preceding the first subsystems level optimization. Then, after each subsystems level (local) optimization set, the system level optimization is performed with new element sizes but with small variations to the original displacements at a limited number of nodal points as the design variables.

### Alternate Subsystems Level Approach

It is expected that the subsystems level optimizations can be further improved through the use of the controlled growth method, developed by Hajela and Sobieszczanski-Sobieski [2], which reduces an optimization to a more efficient analysis with only a slight degradation in accuracy. This method essentially represents an adaptive design variable linking scheme as it is applied in a nonlinear programming based optimization algorithm. It utilizes an effectiveness measure for each design variable and only a single dominant variable controls the growth of all others during the course of an optimization cycle.

## Performance Evaluation

The efficiency of all presented techniques is being evaluated relative to the performance of various optimization approaches [3]: the standard single level optimization approach, where the complete structure is weight minimized under the action of all given constraints by one processor (NAND), and to the performance of the Simultaneous Analysis and Design (SAND) approach [4], where analysis and optimization are combined into a single operation which, for small systems, can increase the accuracy of the solution without a major performance degradation as shown by Striz and Sobieszczanski-Sobieski [5]. The optimization code NPSOL [6], developed at Stanford University and based on the sequential quadratic programming approach, is used for all the optimizations. Both FORTRAN 77 and FORTRAN 90 compilers are utilized to compare their relative performance.

### Simple Cases for Proof of Concept

To date, two initial proof-of-concept models have been developed (Figure 1), a statistically indeterminate fixed-fixed beam under the action of a non-symmetrically placed moment and a dimensionally equivalent truss structure under the same loading condition. The beam model resulted in the same objective function value for the multilevel approach, SAND, and NAND. However, the different orders of magnitude of the displacement coefficients seem to result in ill-conditioned equation systems. To alleviate this problem, the alternate system level approach is presently being investigated, which is expected to result in a more stable system.

Next, the truss structure of similar stiffness will be studied for the same loading condition and results will be compared to the beam case.

## Future Work

More complex two and three-dimensional models are presently being developed to extend the methodology to multidimensional displacement functions.

## Extension to Multidisciplinary Optimization

If efficiency improvements can be shown for the proposed approach in structural optimization as expected, it will be extended to include instability constraints, i.e., buckling, and dynamic constraints, i.e., natural frequencies.

Finally, the methodology will be extended to truly multidisciplinary optimization: by the inclusion of subsystems for the aerodynamics of a realistic aircraft wing in an aeronautics application and for the control of a complex space truss in a space oriented application.

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## References

1. Sobieszczanski-Sobieski, J., Barthelemy, J.F., and Riley, K.M., "Sensitivity of Optimum Solutions of Problem Parameters", **AIAA Journal**, Vol. 20, No. 9, September 1982, pp. 1291 - 1299.

2. Hajela, P., and Sobieszczanski-Sobieski, J., "The Controlled Growth Method - A Tool for Structural Optimization", **AIAA Journal**, Vol. 20, No. 10, October 1982, pp. 1440 - 1441.

3. Lewis, K., and Mistree, F., "On Developing a Taxonomy for Multidisciplinary Design Optimization: A Decision Based Perspective", **Proceedings, First World Congress of Structural and Multidisciplinary Optimization**, Goslar, Germany, May 28 - June 2, 1995.

4. Haftka, R.T., and Gürdal, Z., ***Elements of Structural Optimization***, Kluwer Academic Publishers, Boston, MA, 1992.

5. Striz, A.G., and Sobieszczanski-Sobieski, J., "An Efficient Methodology for the Simultaneous Analysis and Design of Structures", **Proceedings, First World Congress of Structural and Multi-disciplinary Optimization,** Goslar, Germany, May 28 - June 2, 1995.

6. Gill, P.E., Murray, W., Saunders, M.A., and Wright, M.H., ***User's Guide for NPSOL*** *(Version 4.0),* Technical Report SOL 86-2, January 1986, Stanford University, Stanford, California.